

1.
0
1
2
3
4
5
6
7
8

$A[3]$	$A[2]$	$A[1]$	$A[0]$
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0

Increment (A)

```
1 i ← 0
2 while i < length(A) and A[i] = 1
3   do A[i] ← 0
4   i ← i + 1
5 if i < length(A)
6   then A[i] ← 1
```

$$\sum_{i=0}^{\log n} \frac{n}{2^i}$$

$$< \sum_{i=0}^{\infty} \frac{n}{2^i}$$

$$< n \sum_{i=0}^{\infty} \frac{1}{2^i}$$

$$< n \frac{1}{1 - \frac{1}{2}}$$

$$< 2n$$

$n = 2^k$ increment operations

$$T(n) \approx 2n$$

$$T(n)/n \approx 2 = O(1)$$

$$\frac{\cancel{n}}{2} \cdot \frac{\text{size}(T)}{2}$$

$$\underline{2 \cdot \text{size}(T)}$$

$$c_i = \begin{cases} k+1 & \text{if } k=2^j, j=0,1,\dots \\ 1 & \text{otherwise} \end{cases}$$

$$\sum_{i=1}^n 1 + \sum_{k=0}^{\lfloor \lg n \rfloor} 2^k$$

$$= n + \frac{2^{\lfloor \lg n \rfloor + 1} - 1}{2 - 1}$$

$$= n + 2 \cdot 2^{\lfloor \lg n \rfloor} - 1$$

$$< (n + 2 \cdot 2^{\lg n} - 1) + 1$$

$$< 3n = T(n)$$

enqueue(Q, x):
push(S, x)

dequeue(Q):

if S' empty then

while S not empty do

return pop(S') push(S', pop(S))

enqueue: 1
 dequeue: $\begin{cases} \text{non-empty} & S_{i-1}^! : 1 \\ \text{empty} & S_{i-1}^! : |S_{i-1}| \end{cases}$

Use $\phi_i = |S_i|$.

enqueue (Q, x): $a_i = c_i + \phi_i - \phi_{i-1} = 1 + (\phi_{i-1} + 1) - \phi_{i-1} = 2$

dequeue (Q) $\begin{cases} S_{i-1} \text{ empty: } a_i = c_i + \phi_i - \phi_{i-1} = |S_{i-1}| + 1 - 0 = |S_{i-1}| + 1 \\ \text{not-empty: } \phi_i = \phi_{i-1}, \text{ thus } a_i = c_i = 1 \end{cases}$